Riskiness of Sector Dependence In Community Development Financial Institutions

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Abstract

This study addresses the risk exposure of Community Development Financial Institutions' (CDFIs') lending in the real estate sector. CDFI managers can use the findings to estimate how their institution's risk will change with the addition of loans in a particular type of real estate, an important issue during the current cyclical downturn in the housing market. With the Value at Risk indicators presented in this report, stakeholders will have more information on risk to weigh against the expected returns and community impact of CDFIs' lending programs.

Total portfolio Value at Risk of the 180 reporting CDFIs is nearly \$700 million, with \$348 million linked to lending in real estate. Dependence on real estate is concentrated among the largest CDFIs. Average portfolio size is \$17 million and ranges from less than \$0.5 million to more than \$180 million. Many CDFIs specialize in particular types of lending because of missions to fill gaps that are not met by traditional financial institutions. Only one-fourth of the CDFIs have portfolios that are diversified across real estate, business, and consumer lending. For those institutions that lend for a variety of purposes, risk management through diversification is common; more than two-thirds of the CDFIs have a lower Value at Risk when diversification effects are measured. However, the benefits of diversification for those CDFIs are typically small, and in aggregate, there is no risk-reduction benefit from diversification. This unexpected result is due to the largest institutions having portfolio combinations that tend to be risk-increasing.

Introduction

Community development financial institutions (CDFIs) help to address the financial needs of historically underserved, predominantly low-income individuals and communities. CDFIs offer basic financial services, including credit for affordable housing development, and, to a lesser extent, for home purchase mortgages, and debt or equity capital for small and emerging businesses.

On average, mortgage default rates in low-income census tracts are 15 percent higher than in moderate-income ones and 31 percent higher than in middle-income ones (Benjamin et al. 2004). This indication of higher risk, which is likely exacerbated by the cyclical downturn in the real estate sector, potentially threatens CDFIs' ability to accomplish the mission of providing affordable housing. It is not clear to what extent lower-income borrowers and communities have been negatively affected and whether and to what extent CDFIs are affected by the downturn in the real estate industry. This study is an initial step to fill this gap and identify the specific risk exposures in CDFIs' portfolios.

Using newly available data from the CDFI Fund, U.S. Department of the Treasury, and relevant publicly available secondary data, this study addresses the extent to which the CDFIs' portfolios are exposed to risk, focusing on the question of dependence on the real estate sector. Using the Value at Risk technique, we determine the exposure of CDFIs to capital erosion due to cycles in the real estate industry. Several categories of the real estate sector are examined to differentiate exposures among different types of projects, including lending on new homes, mortgage lending, multifamily property development, and retail property development. Further, we consider the extent to which diversification of loan portfolios across real estate and business development lending plays a role in the exposure of the CDFIs.

The findings from this study contribute to the ability of CDFIs to offer financial services in a manner consistent with the risk tolerances of the institutions' stakeholders, and to the policy goal of making sustainable financial services available to underserved individuals and communities. The benefits of Value at Risk indicators for risk measurement and risk communication are demonstrated.

Value at Risk

The risk assessments this research employs are based on the Value at Risk (VaR) technique, which is often implemented as part of the risk measurement and risk communication process in financial institutions. VaR has been adopted in banking, in securities trading, and by major corporations for assessing portfolios. The contribution of this research is the linkage of VaR estimates to a variety of real estate sector indicators as well as business lending and the specific application to portfolios of CDFIs.

We measure the extent to which the missions of community development organizations are at risk due to conditions in the key sectors in which they contribute to community development. The community development mission CDFIs pursue makes them distinct from commercial banks in many regards. However, sustainability with respect to risk-taking in CDFI operations is as important as in commercial banks.

By definition, VaR is the maximum loss over a target horizon such that there is a low, prespecified probability that the actual loss will be larger (Jorion 2001; Holton 2003). That is, VaR is defined as the maximum potential loss in a portfolio value due to adverse market movements, for a given probability and a fixed time horizon. It is a value describing the worst-case outcome.

The risk manager chooses the probability at which to denote the worst-case outcome. For example, if a manager selects 1 percent probability, the dollar value of exposure represents a loss that has a 1 in 100 chance of occurrence. Managers with higher risk tolerance may measure the loss with a higher probability, such as 5 percent. At the higher probability, the estimated dollar value of the loss will be lower, but it is expected to occur more frequently.

Mathematically, we denote by S_t the returns and by VaR_t the amount of dollar loss in terms of returns, for a time horizon of *t*. For a given confidence level *c*, we have VaR_t at the (1 - c) lower-tail percentile (Figure 1) defined as (1) $Pr(S_t < VaR_t(c)) = 1 - c$.

Equation (1) implies that the probability that the financial institution incurs a loss beyond $VaR_t(c)$ equals one minus the confidence level. For example, one bank discloses its daily VaR is \$10 million at the 95 percent confidence level, which represents a 5 percent chance the bank will incur a loss of \$10 million or more over the next day. VaR collapses the distribution of portfolio returns into a single number, which is a readily understandable indicator of risk exposure based on probabilities of loss.

Due to its conceptual simplicity, ease of computation, and ready applicability, VaR has become a standard risk measure for financial risk management. Both in management decision-making within an institution, and in communicating the riskiness of portfolios to external stakeholders, VaR makes a useful contribution.

The origin of VaR can be traced back to 1922 (Mattedi et al. 2004). It became widely used with J. P. Morgan's publication of the RiskMetrics Technical Document in 1995 (J. P. Morgan 1996). Publications for practitioners, by Linsmeier and Pearson (2000) and Jorion (2001), among others, helped popularize this approach to risk management.

Literally hundreds of studies on VaR have appeared in the recent academic literature (Jorion 2002; Manfredo and Leuthold 1999; Mattedi et al. 2004; Yamai and Yoshiba 2005). Many of the prior studies enhance the research basis underlying VaR, which comes from the field of timeseries econometrics. Because multiple sources generate risk exposures, researchers developed methods that account for correlation of the multiple sources, to correctly value the beneficial effects of portfolio diversification. Key drivers of VaR for a global bank, for example, include (Jackson, Maude, and Perraudin 1997): interest rate risk, from exposures related to deposits and cost of funds; equity, due to ownership of publicly traded related or unrelated firms; and foreign exchange, to the extent that deposits or letters of credit involve global sources or uses of funds. Figure 1. Illustration of Value at Risk



Studies applying VaR to the risks faced by financial institutions include Dangl and Lehar (2004) and Jackson, Maude, and Perraudin (1997). These two studies provide an interesting contrast between theoretical approaches found in the former and applied research in the latter. VaR-based regulation of bank capital creates an incentive for banks to reduce risk, in a theoretical model (Dangl and Lehar 2004). The broader implication of this finding is that stakeholders of a financial institution can benefit from wider use of risk-based capital requirements.

There is less agreement on the specific approach a lender should use in making VaR operational. Jackson, Maude, and Perraudin (1997) find that statistical differences do not translate into economically significant differences. They suggest that the accuracy tests in VaR applications not focus on measures of dispersion (standard deviation) as most statisticians do, but instead consider accuracy in predicting the location of the lower tail of the distribution. Historical simulation proves better than parametric approaches in forecasting the lower tail, in Jackson, Maude, and Perraudin's study. They recommend that VaR studies work with a realistic portfolio that includes multiple sources of risk.

Past research results indicate that VaR estimates differ depending on whether they are developed from a simulation-based approach or estimated from a parametric model in which a known statistical form is assumed in representing the risky assets (Christofferson et al. 2001). Perhaps a reason for the difference is that the parametric approach usually involves the assumption of prices being lognormally distributed, a commonly maintained hypothesis among financial market researchers. The flaws in the lognormal assumption are described in the context of VaR by Duffie and Pan (1997), who mention the empirical observation of "fat tails" as one divergence of data from the assumed normal distribution. Also, differences in estimates have been found among various commercial vendors using the same general approach to VaR (Marshall and Siegel 1997).

This study is the first to use the Value at Risk technique to describe the risk profile of CDFIs. A variety of VaR measures are provided to illustrate the choices available to risk managers as they implement the technique.

Mapping Portfolios to Market Risk

Value at Risk depends on the observed value of the risky asset and a probability density function, conditional on information available at time 0. For this application, define the risky asset as the CDFI portfolio. It includes loans for real estate and business development. Initially, the value of the portfolio, V, is known. To predict its future value, it is necessary to forecast because the portfolio is not traded during the time interval (0,1). The outcome of V is based on information available at time 0 (denoted V/v, or V conditional on v).



Decisions regarding the relationship between the assets in the portfolio and market risk are the most critical component of the analysis. We select risk factors that are accessible from secondary data sources but reasonably well represent the risk the institutions are facing. Because our intent is to determine the extent of dependence on real estate, we have identified several asset classes within the real estate sector.

Riskiness of Real Estate Assets

The real estate lending portfolio of CDFIs is classified into four loan types: (1) home purchases and improvement, (2) construction or rehabilitation of single-family homes, (3) construction or rehabilitation of multifamily residences, and (4) commercial real estate projects.

In addition to real estate sector loans, CDFIs lend to projects for business development, consumer credit, and other purposes, which are aggregated as the fifth loan type. The market risk measure associated with each loan type is modeled with variables listed in the right column of Table 1. The risk variables observed in the market relate to the collateral underlying the loan, or, when data are available on traded securities, to the observed movement in securities that are similar in nature to the assets in the portfolio.

Community facility lending is one of the important loan activities of CDFIs by their mission. It is difficult to quantify the risks inherent in lending for community facilities, which include child care facilities and the like. As real estate, it is complicated, and sometimes impossible, to obtain values for these types of public use properties. Without any transaction data, the only source of values is from local appraisal districts or private appraisal reports. In the event the loan defaults and the property is sold, it would likely be valued in the same way as a traditional commercial building, allowing it to be proxied with retail real estate. If the community center is owned or operated by a business entity, the risk associated with lending on the property can be measured by assessing the parent business. Because we are unable to differentiate community facilities from other loans in the data base, they have been aggregated into the fifth asset category.

Loan purpose	Mapping	Risk vectors
Home purchase or rehabilitation	\leftarrow	Price of residential housing
Single-family	\leftarrow	Mortgage REIT stock index ^a
Multifamily	\leftarrow	Multifamily REITs stock index ^a
construction/rehabilitation		5
Commercial property	\leftarrow	Retail REITs stock index ^a
Business, consumer, other	\leftarrow	Russell 2000 small business index

Table 1. Mapping of loan type to risk vectors

^a Value-weighted stock indices based on REIT (real estate investment trust) baskets developed specifically for this study.

Data Sources

Publicly available data on the real estate market are the proxies for the riskiness of the real estate market segments in which CDFIs have lending activities. The distribution of returns for *home purchases* is based on prices of residential housing, collected by the U.S. Bureau of the Census (2007). Monthly new housing prices, both median and average, dating back to 1965 and 1975, respectively, are available. We use the median home price for our analysis because it is less susceptible to the extremes associated with ultra-high-end homes. This information is an excellent indicator of the single-family housing market.

The risks for the other three real estate market segments are represented by monthly prices of various real estate investment trusts (REITs). A REIT is a company that invests in real estate assets, similar to a mutual fund investing in different stocks (Ling and Archer 2006). REITs are highly concentrated in real estate assets, and thus they closely mirror the movements in the specific subsector. A minimum of 75 percent of the value of a REIT's assets must consist of real estate holdings, and at least 75 percent of its gross income must come from real estate assets. In exchange for following these and other rules, REITs are not taxed at the corporate level.

There are three primary types of REITs: equity, mortgage, and hybrid. An equity REIT owns and operates commercial properties, while a mortgage REIT owns mortgage obligations and is in effect a real estate lender. A hybrid REIT invests in both properties and mortgages. Unlike individual real estate investments with infrequent transactions and imperfect information, many REITs are publicly traded and valued every day, so that it is feasible to develop a risk profile based on REITs.

Data on REITs were assembled for as long as the firm has been publicly traded. The vast majority of REIT firms date back 10 years or more, with some going as far back as 1990. We have taken the monthly values of each REIT's stock price and calculated a monthly percentage change.¹

Aggregation of the many REIT companies' returns into a single index involved weighting the price for each firm to create a single weighted average index for each REIT type. The index is weighted by the current share of each firm's assets in the total assets of all selected firms, readjusted annually.

¹ All stock price data retrieved from Yahoo Finance.

The riskiness of CDFI lending for *single-family construction and rehabilitation* is modeled with a basket of eight mortgage REITs.² These REITs purchase and invest in single-family residential home loans. When families apply for a loan, they typically go to a bank or a specialized home lender. The lender will make the loan and then group it with other loans having similar characteristics. This large group of home loans will then be sold into the secondary market, to investors, including mortgage REITs. The money the banks receive from investors for the purchase goes toward funding new loans, and the cycle repeats itself. In essence, mortgage REITs provide the bank with capital to continue to make loans.

The REITs included in the single-family index all invest in home loans of various types. Many invest in fixed-rate, floating-rate, and adjustable-rate loans, while some focus on the floatingand adjustable-rate varieties that have been commonly used by lower-credit-quality borrowers. Many of the firms invest in the highest-quality mortgages, considered to be the safest, as well as riskier ones that offer higher returns. With the current mortgage crisis, many of these so-called safe investments are being repriced with higher risk associated with them. In essence, these mortgage REITs are the true source of capital used to make home loans, and they correspond well to the loans made and held by banks. While the CDFIs may not sell their loans into the secondary market, the same economic forces act upon the value of the loans being held in their portfolios.

For loans to *multifamily housing construction and rehabilitation* projects, risk is modeled with a basket of 10 multifamily equity REITs.³ These firms own multifamily properties and receive their income, and subsequent valuations, based upon the real estate's performance. The firms develop, redevelop, acquire, own, and operate all types of multifamily dwellings across the United States. These firms provide excellent insight into the stability or riskiness of the multifamily housing sector.

Some of the specific REITs in the multifamily basket, such as Apartment Investment and Management Company (AIV), and Home Properties Inc. (HIC), specialize in properties that are most likely served by CDFIs and similar institutions. AIV owns, redevelops, and manages many "affordable apartments" that qualify for subsidies from the Department of Housing and Urban Development. HIC specializes in low-end to mid-price properties. Many of these apartments are 20–40 years old and might not be easily financed by traditional banks. These types of lower-priced and traditionally underserved assets are at the heart of the CDFI mission, and the values reflected by the group of multifamily REITs as a whole are a good proxy for the values of the multifamily loans in a CDFI's portfolio.

Commercial real estate REITs are the basis for the risk associated with CDFI lending for *commercial properties construction and rehabilitation*. Commercial REITs are separated into two groups, office properties and retail properties. Office REITs are invested in primarily urban,

² The eight mortgage REITs are Thornburg Mortgage Inc. (TMA), Annaly Capital Management Inc. (NLY), Impact Mortgage Holdings Inc. (IMH), Friedman Billing Ramsey Group Inc. (FBR), MFA Mortgage Investments Inc. (MFA), Anworth Mortgage Asset Corp. (ANH), Capstead Mortgage Corp. (CMO), and Novaster Financial Inc. (NFI).

³ The 10 multifamily REITs are Equity Residential (EQR), Apartment Investment and Management Co. (AIV), AvalonBay Communities Inc. (AVB), Camden Property Trust (CPT), United Dominion Realty Trust, Inc. (UDR), Home Properties of New York, Inc. (HME), BRE Properties Inc. (BRE), Essex Property Trust (ESS), Post Properties Inc. (PPS), and Mid-America Apartment Communities Inc. (MAA).

high-rise office buildings. This property type has very little in common with CDFI lending, so we have excluded commercial office REITs from the index. Retail REITs invest in much smaller and less expensive local properties that are similar to those property types to which a CDFI would be lending. Therefore, we have used only retail commercial REITs in our analysis.

The retail REIT basket consists of eight firms.⁴ These firms develop, redevelop, purchase, own, and operate retail commercial properties ranging from neighborhood shopping centers to individual commercial buildings. Once again, this group includes firms that own real estate around the country and is therefore geographically diverse. All these property types could be subject to CDFI loans, and the risk being carried on the CDFI balance sheet is mimicked by the risk and volatility of these retail commercial REITs' prices.

In addition to real estate lending, CDFIs have a presence in supporting projects in consumer credit and in lending to businesses that provide employment and development opportunities. Community facilities, such as child care centers, are included in this aggregate other category. The risk profile of the combined *non-real-estate portions of the portfolio* is modeled with the Russell 2000 index. This index is published by Russell Investments to measure the performance of the small-cap segment of U.S. publicly traded equities. The index is composed of approximately 2,000 of the smallest public companies, and it is reformulated every year to ensure its accuracy. The Russell 2000 is the most widely accepted and used measure for the performance of small business in the United States. This index provides the best way to obtain reliable and accurate information on small companies similar to those served by CDFIs.

Analysis of the Risk Factors

Each asset that will proxy for a portion of the CDFI portfolios is examined in terms of summary statistics and graphical analysis. The price levels of the assets and their rates of return or movement over a month are analyzed separately. Figure 2 shows the time series of monthly closing data for the median price of new homes, the mortgage REIT basket, the multifamily REIT basket, the retail REIT basket, and the Russell 2000 index, which are used as proxies for loan types 1 through 5, respectively. The common price trend in nominal terms has been upward over the long term, but new home and REIT prices have declined in recent months. The series are updated as of December 2007 or October 2007, as available.

The price levels are converted to real rates of return with an adjustment for inflation. We use PCEPILFE (Personal Consumption Expenditures: Chain-Type Price Index Less Food and Energy) as the price deflator. The index is from the Bureau of Economic Analysis of the U.S. Department of Commerce. We also adjust for dividends paid in computing rates of return on the mortgage REIT basket, the multifamily REIT basket, and the retail REIT basket.

After the adjustments, we calculate the monthly rate of return (simple return) based on the value in the current month and the month before. Figure 3 illustrates the rates of return for new homes, the mortgage REIT basket, the multifamily REIT basket, the retail REIT basket, and the Russell

⁴ Retail REITs are Developers Diversified Realty Corporation (DDR), Kimco Realty Corporation (KIM), Weingarten Realty Investors (WRI), Regency Centers Corporation (REG), Realty Income Corporation (O), National Retail Properties, Inc. (MMN), Equity One, Inc. (EQY), and Acadia Realty Trust (AKR).

2000 index. Relatively large downward spikes in the series of returns occur from time to time, and these translate into large short-term losses on the corresponding asset. From the monthly rates of return over time, one can investigate a variety of measures of riskiness. Standard deviation is the most commonly used indicator of variability in finance. Mortgage lending and business lending have the highest standard deviation among the loan types (Table 2). The range (difference between lowest and highest rates of return in the series) is also largest for the mortgage REIT basket (0.63) and the Russell 2000 (0.47). Interestingly, the higher risk for the business lending was not associated with higher average returns. The business risk indicator has a relatively low mean return (8 percent), along with high risk. Mortgage lending was consistent with a risk–return tradeoff, having the second highest average monthly rate of return, 16 percent on average, as well as the highest risk (standard deviation of 8.6 percent).

New homes are the least risky, with a standard deviation of 3.6 percent and range of 20 percent. On average since 1966, new home prices rose 0.5 percent in a month. The monthly growth in new home prices was much more rapid during the 2000s than earlier in the period.

VaR indicators are derived from the lower tail of the probability distributions; therefore, it is interesting to examine the time series to identify the periods during which the largest losses occurred (Figure 3). The rate-of-return graphs are a good visual representation of the volatility of the index prices and REIT baskets that are used as proxies for the loan types. Multifamily real estate risks appear to be on the rise, and time-related growth in risk is even more apparent for retail REITs. In spite of an apparent higher risk of negative returns for multifamily retail REITs, the range of losses has been consistently lower than for the higher-risk mortgage REITs. Infrequent downward spikes in the returns series contribute disproportionately to the risk exposure. For example, mortgage REITs' greatest variability was in the 1990s, the last major downturn in the real estate sector.

While the recent developments in the housing market represent a significant reduction in home prices, historical price series remain an appropriate baseline for representing risk. The historical series used for this analysis includes other instances of significant home price decline such as the time period from the mid-1980s until the early 1990s, as well as the leading edge of the current declines. These periods have been included in the calculations of monthly loss probabilities for home purchase or rehabilitation and are reflected in our calculations of VaR.

From July 2007, when the subprime mortgage problems first came to light, through March 2008, new home prices have declined in four months and risen in the other five. The rates of return from November 2007 through March 2008 mirror other periods of housing decline and are not out of the ordinary (Table 3). However, of the four months of decline, two are historically significant. While these losses of 8.64 percent in October and 8.59 percent in December are significant, they do not reach the depths of the worst-case recorded monthly loss of 9.4 percent (Table 2). Current market conditions, while serious, would not be outside the realm of possibility to an analyst using VaR from the historical series at the start of 2007.



Figure 2. Nominal prices for real estate and business assets, 1965-2007

(e) Type 5: Business, consumer, and other

Figure 3. Monthly rates of return on real estate and business assets, adjusted for dividends and inflation, 1965-2007



(e) Type 5: Business and consumer loans

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	Type 1: Home	Type 2: Single- family constr /	Type 3: Multifamily	Type 4: Commercial	Type 5: Business
	purchase or rehabilitation	rehab.	constr./rehab	Commercial	consumer, and other
	Median price of new home sale	Mortgage REIT	Multifamily REIT	Retail REIT	Russell 2000
Time Range	1965:2-	1990:2-	1990:2-	1990:2-	1987:10-
-	2007:10	2007:12	2007:12	2007:12	2007:12
No. of Obs.	513	215	215	215	243
Percentage					
Mean	0.005	0.016	0.017	0.015	0.008
Minimum	-0.094	-0.307	-0.132	-0.166	-0.301
Maximum	0.106	0.239	0.141	0.136	0.164
Std. deviation	0.036	0.086	0.047	0.045	0.055
Skewness	-0.019	-0.418	-0.179	-0.224	-1.127
Kurtosis	2.901	5.241	3.329	4.348	7.530
1% percentile	-0.074	-0.245	-0.101	-0.098	-0.152
5% percentile	-0.059	-0.116	-0.064	-0.056	-0.073

Table 2. Summary statistics of the distribution of rates of return on real estate and business assets based on frequency histogram of series, adjusted for dividends and inflation

Note: The percentiles of the frequency histogram differ from the tail loss estimate based on alternative empirical procedures shown later in the paper.

Table 3. Median price of new single-family homes, monthly 2007-2008

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Month	Median	Change	Return (%)	
Nov-07	\$249,100	\$31,300	14.37%	
Dec-07	\$227,700	(\$21,400)	-8.59%	
Jan-08	\$232,900	\$5,200	2.28%	
Feb-08	\$244,200	\$11,300	4.85%	
Mar-08	\$227,600	(\$16,600)	-6.80%	

Source: U.S. Bureau of the Census.

The worst-case outcome for business risk occurred some time ago. The Russell 2000 rate-ofreturn series contains approximately six to eight unusually large negative returns. The first of these large negative returns occurs at the beginning of the series in October 1987, the month Black Monday occurred. Thus, as shown by the Russell 2000 graph (Figure 3), one extremely negative return has significant impact on the range (and the lower tail of the returns distribution); these effects increase the VaR. As a series based on publicly traded firms, it is possible that the Russell index understates the riskiness of small business clientele served by CDFIs.

In addition to the common statistics that measure risk and reward, the shapes of the probability distributions for the risky assets were considered in detail, so that the empirically derived risk estimate will take into account all features of the risk. The two additional shape parameters considered are skewness and kurtosis. Skewness indicates the asymmetry of the distribution. All five loan types have a negative skewness (Table 2), which suggests that the left tail is longer and the mass of the distribution is concentrated on the right. The significance of left-skewness for VaR calculations is that a potential loss is quite large, even if it is a rare event. The greatest skewness among the risk factors is found for the small business returns (Russell 2000).

All five loan types have a positive kurtosis, i.e., the distributions of the rate of return of these five types are leptokurtic. In terms of shape, a leptokurtic distribution has a more acute peak around the mean and fat tails. For example, type 5 (business and consumer loans represented by Russell 2000) has the highest kurtosis, which suggests that infrequent extreme deviations contribute more to its variance, as indicated by the fat left tail.

The skewness and leptokurtosis findings support the decision not to assume a normal distribution for asset prices or returns. This issue is addressed in more detail in Appendix A. Instead, an empirical kernel estimator procedure is used to model the risk profile.

The interaction of the returns on risky assets is a final important feature in development of the risk profile. The correlation between five loan type proxies is informative as to the potential for risk reduction through portfolio diversification (Table 4). Assets in new homes are negatively correlated with the other four asset types. Negative correlations are consistent with portfolio risk being smaller than the risk when measured without taking portfolio effects into account. When assets move in opposite directions at a given point in time, risk is lower because gains in one area offset losses in another. However, the negative correlations across the asset types are small, not exceeding 0.05 for any two asset types. Therefore, to the extent that there is a risk reduction associated with diversification across loan types, it is expected to be small.

There are sizable positive correlations between mortgage assets and multifamily housing and retail real estate (0.19–0.22). Multifamily projects are highly correlated (0.46) with returns from retail development, the largest correlation among asset types. Given positive correlations, less risk reduction associated with the portfolio returns is expected. Indeed, it is possible that positive correlations lead to higher risk from a portfolio-based estimate that takes into account all interactions, compared with a procedure that overlooks the interactions between risk factors.

	Type 1:	Type 2:	Type 3:	Type 4:	Type 5:
	Home	Single-family	Multifamily	Commercial	Business and
	purchase or	constr./	constr./rehab		consumer loans
_	rehabilitation	rehab.			
	Median price	Mortgage	Multifamily	Retail REIT	Russell 2000
	of new home	REIT	REIT		
	sale				
Covariances					
Homes	0.0015226	-0.0000555	-0.0001326	-0.00010393	-0.00011577
Mortgages	-0.0000555	0.00725296	0.0010558	0.00097963	0.00165068
Multifamily	-0.0001326	0.00105577	0.0020740	0.00134386	0.00101012
Retail	-0.0001039	0.00097963	0.0013439	0.00194089	0.00079284
Business	-0.0001158	0.00165068	0.0010101	0.000792844	0.00265738
Correlations					
Homes	1.00000	-0.01670	-0.07461	-0.06046	-0.05756
Mortgages	-0.01670	1.00000	0.27221	0.26110	0.37599
Multifamily	-0.07461	0.27221	1.00000	0.66980	0.43027
Retail	-0.06046	0.26110	0.66980	1.00000	0.34911
Business	-0.05756	0.37599	0.43027	0.34911	1.00000

Table 4. Covariances and correlations of the rates of return of real estate and business assets, 1990-2007

Note: Dates included from Feb. 1990-Dec. 2007.

Empirical Density Estimation

The goal of our research is to describe risks with a statistical procedure that reflects the historically observed data. To the extent possible, the procedure allows the data to be reflected in the technical model, with a technique that is data driven preferred to one based on theory alone. Both frequency histograms and kernel density estimation are used to estimate the empirical densities of monthly rate of returns.

Histogram plots are commonly used to graphically display frequencies, or the relative probability, of a random variable. Intuitively, histograms group observations together into bins. The histogram density estimator can be valid (asymptotically consistent), but less smooth than an estimate obtained from a parametric approach that assumes a particular mathematical form of the distribution, or from a nonparametric approach that empirically fits data points.

The kernel density estimator is an improvement over a basic histogram plot. Procedurally, one can think of the results from a kernel estimator as a histogram with the addition of small "bumps" at each observation. The bumps represent the fact that each observation is a realization from a distribution that can be estimated in small parts. The kernel density estimator consists of a "sum of bumps" and is smoother than a frequency histogram. It is well known that the choice of kernel functions has a minimal impact on the estimated probability distribution, but the bandwidth is critical in estimating the probability density function. Small values of the bandwidth lead to very spiky estimates (not much smoothing) while larger values can lead to oversmoothing. A common method to determine the optimal bandwidth is to use the bandwidth that minimizes the asymptotic mean integrated square error. The technical details of the kernel

density estimation procedure are presented in Appendix B. We use the Gaussian kernel function, and the bandwidth is selected with the method of Sheather and Jones (1991).

Figure 4 shows the results of the empirical derivation of the probabilities of loss associated with each type of loan in the CDFI portfolio. The horizontal x-axes represent the rates of return, and the vertical y-axes represent the associated relative probability. The curve associated with each histogram plot is the kernel density estimation for the rate of return for that loan type proxy. The procedure clearly captures the asymmetry of the data and contains appropriate detail about the tails.

The left tail of each distribution (Table 5) is the main concern because this is where large portfolio losses occur. An unlikely, but damaging, loss outcome is defined by examining percentiles of the distribution. Percentiles are areas below a defined point on a probability density function. A 0.01 percentile signifies that only 1 percent of the time, an observation falls to the left (below) the range in the left tail. The boundary of the percentile is termed a tail loss percentage or a percentile of the rate of return distribution. This tail loss number is the key input into VaR calculations.

For example, consider the riskiness of lending to businesses. Selecting the loss from a small area that defines a 1 percent chance of occurrence, also called tail loss or 0.01 percentile, the associated tail loss is 34.3 percent for a month. The estimated potential loss over a month is more than one-third of the initial portfolio value. Higher risk tolerances (5 percent probability) are associated with lower loss levels (32.1 percent).

Three of the four real estate asset types are less risky than the business lending category. Home purchase and improvement is the least-risky type of lending. The lowest 1 percent of the housing price distribution describes a possible loss of 12.4 percent over a month. Multifamily housing development is next in terms of riskiness, at a 16.8 percent tail loss estimate. Commercial real estate development has an exposure estimate of 20.3 percent. The most risky segment of the real estate market is the mortgage market, at 36.5 percent loss exposure for the lowest percentile.

The rates of return for the mortgage REIT basket, retail REIT basket, and Russell 2000 have a significant fat tail on the left side of the empirical distribution (Figure 4). The finding is consistent with the skewness of these three proxies reported in Table 2—the Russell 2000 has the largest left-skewness, followed by the mortgage REIT basket, and then the retail REIT basket. The significance of left-skewness for VaR calculations is that a potential loss is quite large, even if it is a rare event. Thus, the finding of the fat left tails of these three proxies concurs with the tail loss reported in Table 5—the mortgage REIT basket has the largest tail loss, followed by the Russell 2000, and then the retail REIT basket.

Figure 4. Estimated densities of rates of return, real estate and business assets, 1965-2007

New Home Sales Median Price (1965:1-.2007:10)



Multifamily REITs (1990:2--2007:12)



Mortgage REITs (1990:2--2007:12)



Retail REITs (1990:2--2007:12)



Russell 2000 (1987:9--2007:12)



Table 5. Estimated risk exposure (tail loss percentages) on real estate and business assets, 1965-2007

Loan type	Rate of return index	Percentil	es of rate of	f return
		1%	5%	10%
Home improvement and purchase	Census housing prices	0.12424	0.11399	0.10118
Single-family RE Rehab. and Constr.	Mortgage REIT basket	0.36480	0.33773	0.29936
Multifamily RE Rehab. and Constr.	Multifamily REIT basket	0.16818	0.15446	0.13729
Commercial RE Rehab. and Constr.	Retail REIT basket	0.20321	0.18815	0.16933
Business	Russell 2000 stock index	0.34324	0.32128	0.29383

U.S. Department of the Treasury, CDFI Fund - Research Initiative

Estimated empirical probability density function developed from monthly rates of return, adjusted for dividends and inflation.

CDFI Portfolios

In total, the loan portfolio of the CDFIs exceeds \$3 billion according to the Institution Level Report, 2005 (U.S. Department of the Treasury 2007, Community Investment Impact System [CIIS] data). The 180 CDFIs that provide portfolio information for 2005 constitute a relatively small share of the 750 CDFIs that were certified in that year. The reporting institutions encompass a wide range of sizes. The average portfolio amount is \$17 million (Table 6), although the median is much smaller, at \$5.4 million, indicating that there are many small entities in the dataset. The institutions' loan portfolios range from less than \$0.5 million to more than \$180 million. Cumulative numbers by size category (Table 7) indicate that 100 institutions lend \$5 million or less. The 10 largest CDFIs each have more than \$80 million in loans outstanding.

Specialization is fairly common among the CDFIs. More than 40 percent of the CDFIs (73) have their entire portfolio in one loan type, as loans are classified in this study (Figure 5). Sixty of these 73 CDFIs are focused on lending to businesses (type 5) and probably diversify the business loans across industries. The number of loan types is statistically related to size of the CDFI, but to a very small extent.

Table 6. Descriptive	statistics for CDFI p	ortfolios in million dol	lars, 2005	
Mean	16.75	Minimum	0.0010	
Median	5.40	Maximum	180.89	
		Standard		
Range	180.89	deviation	30.88	
Skewness	0.00	Standard error	2.30	
Total	3,015.73			

Source: Institution Level Report, 2005.

\$ millions in portfolio	Number of institutions	\$ millions in portfolio	Number of institutions
0.5 or less	17	70 or less	169
5 or less	86	80 or less	169
10 or less	114	90 or less	172
20 or less	143	100 or less	173
30 or less	155	125 or less	175
40 or less	158	175 or less	177
50 or less	165	200 or less	180
60 or less	168		

Table 7. Cumulative number of CDFIs by portfolio value, 2005

The 107 institutions that have more than one type of loan have a varying selection of loans across real estate market segments or across real estate and business lending (Table 8). Most of the institutions reporting in 2005 have real estate as an important segment of their lending

activities (Figure 6). Forty-four CDFIs have up to 50 percent of the portfolio in real estate assets and are considered *diversified* across real estate, business, and consumer lending. Around one-third of the CDFIs (65) have a strong focus on real estate activities, with between two-thirds and 100 percent of the portfolio in the property sectors. The institutions having 50 percent to 67 percent of the loans in real estate are classified as *specialized in real estate*, while 29 percent of the CDFIs have all of their loans in real estate projects. Sixty of the 180 CDFIs (one-third of the institutions) have no exposure to real estate at all and lend only to business projects or consumers.

Loan portfolio assets differ across institutions by other classifications as well, including rural vs. urban, women-owned vs. others,⁵ minority-owned vs. others, faith-based vs. others, and financial institution types (Figure 7). There are more CDFIs in rural areas, but the average asset size of a CDFI in rural areas does not differ that much from those elsewhere (Table 9).

Portfolios differ among different financial institution types. For example, on average, credit unions, venture capital funds, and loan funds tend to have smaller portfolios than banks and thrifts.



Figure 5. Number of loan types in CDFIs' portfolios

⁵ The term "women-owned" refers to the designation "women-owned or controlled" as reported by CDFIs in the CIIS data. The term "minority-owned" refers to the designation "minority-owned or controlled" in the CIIS. These terms are not references to the populations served by these CDFIs.





Table 8.	CDFI	portfolio	bv	loan	type.	2005
			~ _		- / 7	

Loan type	Variable name	Frequency (% of all loans)	Value in \$ millions (% of all loans)	Mean (\$) [std deviation]
Type 1		,		
~ I		1,122	74	66,375
Home improvement	HOMEIMP	(3.53)	(2.47)	[854,207]
		4,920	273	55,401
Home purchase	HOMEPURCH	(15.47)	9.04)	[507,797)
•		6,042	347	57,439
Subtotal		(19.00)	(11.51)	[587,695]
Type 2				
		162	58	360,337
RE Rehab, single-fam.	RERHSINGLE	(0.51)	(1.94)	[1,689,429]
		389	142	365,157
RE Constr., single-fam.	RECOSINGLE	(1.22)	(4.71)	[1,948,484]
		551	200	363,740
Subtotal		(1.73)	(6.64)	[1,872,149]
Type 3				
RE Constr.,		366	222	607,591
Multifamily	RECOMULTI	(1.15)	(7.37)	[2,052,086]
RE Rehab.,		470	447	950,806
Multifamily	RERHMULTI	(1.48)	(14.82)	[7,563,905]
		836	669	800,547
Subtotal		(2.63)	(2.21)	[5,831,364]
Type 4				
RE Constr.,		293	410	1,400,790
commercial	RECOCOM	(092)	(13.61)	[7,545,948]
RE Rehab.,		400	174	434,888
commercial	RERHCOM	(1.26)	(5.77)	[1,120,726]
		693	584	843,271
Subtotal		(2.18)	(19.38)	[4,997,948]
Type 5				
		1,825	285	155,976
U.S. Department of the Tre Business-working	easury, CDFTFund – F	Research Initiative 4,346	(9.44) 317	[1,482,804] 72,941
capital	BUSWORKCAP	(13.67)	(10.51)	[618,171]



Figure 7. Portfolio value of CDFIs, by type of institution, 2005

Table 9. Portfolio of CDFIs, by location of CDFIs

Number of		
observations	104	62
	Portfolio value (\$)	
Mean	16,527,685	15,684,485
Median	6,018,297	4,692,234
Range	172,559,386	179,207,730
Skewness	3	4
Minimum	1,049	9,270
Maximum	172,560,435	179,217,000
Standard		
deviation	26,870,510	32,954,023
Total	1,718,879,251	972,438,063
Minimum Maximum Standard deviation Total	1,049 172,560,435 26,870,510 1,718,879,251	9,270 179,217,000 32,954,023 972,438,063

Procedures for Estimating Exposure with Value at Risk

Value at Risk (VaR) is calculated from the lower tail of a probability distribution of returns. The percentage loss associated with a small area in the tail is multiplied by the book value of the loans in the portfolio, yielding a VaR estimate in dollars. This single number, for each institution, is a means for communicating risk exposure that is easily understood. Based on the VaR estimate, the risk management process can be altered, if necessary.

Three alternative methods to estimate the risk indicator according to the CDFIs' portfolios are reported in this section—undiversified, or standalone, VaR; diversified portfolio VaR; and independent portfolio VaR.

Undiversified Value at Risk

Undiversified, or standalone, VaR, is the simplest to calculate but overlooks potential benefits that may be obtained from assembling a diversified portfolio. To calculate undiversified VaR, identify tail loss amounts for each asset in the institution's portfolio and multiply by the value of loans in the category, then sum for each institution. The calculation for undiversified VaR is
(2) $VaR_{1} = \sum X_{k}z_{k},$

where X_k is the dollar value of loans in each of k types, and z is the tail loss (in decimal form, representing a chosen percentile) for each of the k asset types. Undiversified VaR is an indicator for the possible worst-case situation, in that potentially beneficial interactions among the random factors are not considered.

Sustainability of institutions that provide financial services to low-income groups is a primary concern of this research. Pioneers in financial theory have shown that diversification of investments offers advantages in managing risks (Markowitz 1952) for an individual investor. The well-known adage "Don't put all your eggs in one basket" makes the same point. While the ability of CDFIs to diversify their loan portfolios is constrained by the specific mission, location, and nonprofit status of the various institutions, the effect on the mission and sustainability of

portfolio diversification advantages should be understood. Therefore, a procedure for accounting for diversification is developed in this study, as outlined in the next section.

Mathematical Presentation of Portfolio Diversification

It has long been recognized that risk profiles differ when the interactions of multiple random variables are taken into account. Hence, as a complement to the standalone VaR estimates, risk exposure is estimated in relation to the entire portfolio. In a portfolio setting, the interaction of historical random variables is critical information, because gains on certain assets may be useful in compensating for losses on other assets.

Following portfolio theory, define the future portfolio value as a probability distribution of possible future asset values, rather than a single observation. Mathematically, the future portfolio value is written as $V^1 = \omega S^1$. The value of loans in each category (ω) is known and assumed not to change over the forecast horizon. *S* is the vector of risky assets, and the superscript 1 indicates a predicted value. Define σ as the standard deviation (or riskiness) of V^I , and Σ as the covariance matrix, which measures the interactions between the elements of S^I . Both of these volatility parameters are understood to depend on information available at time 0. Because of the linear relationship between the portfolio value and its components, the single parameter to represent the portfolio risk (its standard deviation) is estimated from this formula:

(3)
$$\sigma = \sqrt{\omega \Sigma \omega'}$$

Elements in the matrix Σ are estimated from historical information. The structure of the covariance matrix is allowed to vary across different portfolio VaR calculations, so that comparisons can be made to isolate the impact of asset interactions. Diversification of the portfolio is accounted for in two ways: one that accounts for all the interactions of random factors, which will be called "diversified portfolio VaR," and another that maintains the assumption that assets are independent of each other, termed "independent portfolio VaR." That is, correlations across assets are assumed to be zero in the independent portfolio VaR. Specifically, we use a full five-variable covariance matrix for the calculation of portfolio VaR, and a diagonal covariance matrix for an estimate of VaR that does not take the portfolio interactions into account.

Consider a portfolio with two different assets, X_1 and X_2 . The covariance matrix of the rate of

return on these two assets is given by $\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$, where σ_i^2 is the variance of the rate of

return on asset *i* and σ_{12} is the covariance of the rates of return between two assets.

Standalone (Undiversified) VaR

The standalone VaR is simply calculated as $VAR_i = \alpha \sigma_i X_i$. The parameter α is the percentile of the probability density function associated with a specified riskiness defined by the tail loss area. With multiple variables, undiversified VaR is

(4)
$$VAR_{p}^{S} = \alpha \begin{bmatrix} X_{1} & X_{2} \end{bmatrix} \begin{bmatrix} \overline{\sigma}_{1} \\ \sigma_{1} \end{bmatrix} = \alpha (X_{1}\sigma_{1} + X_{2}\sigma_{2}).$$

Diversified Portfolio VaR

The diversified VaR takes portfolio diversification into account completely. That is, both the volatility of its own assets as well as the dependence between assets are factored in when calculating the diversified VaR. Formally, the formula to calculate the diversified VaR is:

(5)
$$VAR_{p}^{D} = \alpha \sqrt{\left[X_{1} \ X_{2}\right]} \Sigma \begin{bmatrix} X_{1} \\ X_{2} \end{bmatrix}} = \alpha \sqrt{X_{1}^{2}\sigma_{1}^{2} + 2X_{1}X_{2}\sigma_{12} + X_{2}^{2}\sigma_{2}^{2}}.$$

The significant new elements in equation (5) compared with equation (6) are in **bold** font. They are the components involving interaction of the two assets.

Independent Portfolio VAR

The independent portfolio VAR uses the multiple variables in the portfolio, but assumes no correlation between the rates of return on different assets. Thus, the off-diagonal entries in Σ are assumed to be zeros. Algebraically, the independent portfolio VAR is

(6)
$$VAR_{p}^{I} = \alpha \sqrt{\begin{bmatrix} X_{1} & X_{2} \end{bmatrix} \begin{bmatrix} \overline{\sigma}_{1}^{2} & 0 \\ 0 & \sigma_{2}^{2} \end{bmatrix} \begin{bmatrix} X_{1} \\ X_{2} \end{bmatrix}} = \alpha \sqrt{X_{1}^{2} \sigma_{1}^{2} + X_{2}^{2} \sigma_{2}^{2}}.$$

To isolate the difference in the three VaR measures, rewrite equation (4) for standalone VaR into

(7)
$$VAR_{p}^{s} = \alpha \left[X_{1} \ X_{2} \right] \begin{bmatrix} \sigma_{1} \\ \sigma_{2} \end{bmatrix} = \alpha \left(X_{1}\sigma_{1} + X_{2}\sigma_{2} \right) = \sqrt{X_{1}^{2}\sigma_{1}^{2} + 2X_{1}X_{2}\sigma_{1}\sigma_{2} + X_{2}^{2}\sigma_{2}^{2}}$$

Compare equations (5) and (7) to identify the difference between these three measures of VaR for a portfolio of two assets. The differences are highlighted in bold. The standalone portfolio VAR is always higher than the independent portfolio VaR measure. Thus, the simplest VaR calculation method leads to a larger indicator of risk, or the worst case.

The advantage of independent portfolio VaR and diversified portfolio VaR is that the formulas take the benefits of diversification into account. VaR will be smaller, consistently, when portfolio effects are included. If the rates of return of two assets are negatively correlated, we expect the diversified portfolio VaR to be even smaller.

The difference between diversified VaR and undiversified (independent portfolio) VaR represents the benefits from diversification. As long as the correlation coefficient between the

rates of the returns of two assets is smaller than one, i.e., $\frac{\sigma_{12}}{\sigma_1 \sigma_2} < 1$, the diversified VaR is always

lower than the undiversified VaR, and hence, diversification leads to a lower VaR indicator. There are cases, however, in which asset returns are not negatively correlated. In a portfolio with multiple assets, the possibility of positive interactions of different portions of the portfolio can result in risk exposure measures that are larger when portfolio diversification is taken into account.

Example of Portfolio Effects on Risk

As an example, consider the portfolio of an actual CDFI that has loans in three categories: home purchases (type 1), multifamily residential development (type 3), and business/consumer (type 5). The vast majority of loans (over 80 percent) in the portfolio are in type 5, which are high-risk. Around 15 percent of the portfolio is in single-family homes (type 1), which are relatively less

risky and tend to move in the opposite direction of the business loan category. Negative correlation also is present between the two real estate types in this portfolio. These relationships indicate the potential for diversification to have advantages for the institution.

The diversified portfolio VaR is \$16.04 million, contrasted with the independent portfolio VaR, at \$16.27 million. While the risk estimates for this single institution vary due to diversification, the difference is fairly small. Diversification benefits, or lack of benefits, is a case-specific question that depends on the components of the portfolio for each institution. The estimates of portfolio VaR for each of the 180 respondent CDFIs were obtained from a simulation model that is used to identify the diversification effects for each institution. The methods of the simulation are described in the next section, followed by the findings on risk for the CDFIs.

Simulation of Portfolio VaR

The basic idea of the simulation model is to define a sample based on the observed historical observations, and then construct a set of possible realizations that reflect the historical record, while incorporating uncertainty, including interactions of the random variables. The approach is selected so that the heavy tails in the observed distributions are retained when the risk measure is calculated.

The simulated set of realizations that reflects the historical record is generated from a random process. Pseudo-random numbers for each asset type are generated from a standard normal distribution (standard normal deviates) and then correlated to one another based on the historically observed correlation. These correlated standard normal deviates are transformed into correlated uniformly distributed numbers (correlated uniform deviates). Each correlated uniform deviate is used to retrieve values from the empirical returns distribution that is associated with its corresponding asset type.

The simulation algorithm is implemented in R (computing package) and is used to obtain an unbiased random series for each of the five loan types. A common seed value of the random number generator is used for the alternative simulations to ensure comparability.

The pseudo-random numbers are used to construct realizations of each random variable, accounting for covariances across the asset types. The draws from the returns distributions for each asset type are aligned with the institutions' portfolio data to create a probability density function of portfolio returns for each institution. From this probability density, the Value at Risk estimate for each CDFI is calculated.

Results

The total Value at Risk for all 180 CDFIs is approximately \$700 million (Table 10). The interpretation of this estimate is that there is a 1 percent chance that the industry loss will reach as high as \$700 million or more in a month. Industry-level exposure is \$612 million or more, with a 5 percent chance of occurrence. The influence of portfolio diversification is considered in this estimate.

Value at Risk is decomposed into risk associated with real estate, compared with other types of loans (Table 10). The total VaR in real estate is \$350 million (at the 1 percent probability). The

mean of CDFIs' real estate VaR is \$1.9 million, quite close to the mean risk due to business and consumer lending, indicating that there is no major difference on average in exposure to real estate compared with other types of loans. There is greater dispersion of real estate VaR among the CDFIs, indicated by the high standard deviation (\$5.33 million). The maximum VaR in real estate among the 180 CDFIs is nearly \$40 million, which is much greater than the maximum in other types of loans. Thus, it is apparent that while real estate exposure does not dominate the aggregate risk of CDFIs, individual institutions have considerable risk due to such exposures. The extent of riskiness due to dependence on real estate is further illustrated in Figure 8. The height of the bars is total VaR in dollars, with one bar representing each CDFI. The bottom portion of the bar (white) is real estate VaR in dollars for that CDFI. The lower portion (black) of the bar represents VaR in business and consumer loans. The bars are arranged by size of the CDFI loan portfolio. There is a clear focus on real estate among the largest CDFIs. The larger estimates of exposure to real estate, industrywide, is predominantly due to the activities of these few, very large CDFIs. It is also interesting to note that the largest CDFI is not the riskiest, as measured by VaR.

The comparison by urban and rural locations of CDFIs does not show major differences in terms of their VaR (Table 11).

	Sum for 180				
	CDFIs	Mean	Std dev	Min	Max
Portfolio VaR					
Total 1%	705	3.91	6.67	0.00^{a}	36.61
Total 5%	612	3.40	5.73	0.00^{a}	30.62
Undiversified VaR					
Business/consumer lending,					
1%	415	2.32	4.16	0.00	31.84
Business/consumer lending,					
5%	387	2.17	3.89	0.00	29.80
Real estate lending, 1%	350	1.93	5.33	0.00	39.61
Real estate lending, 5%	324	1.78	4.91	0.00	36.43

Table 10. Summary statistics on	VaR estimates, in million dollars,	, for CDFIs, total and by loan type
The for Summing Statistics on	are obtimutos, in ministration domais,	, 101 021 15, total and cy loan type

^aLess than .0001 million.

Figure 8. Value at Risk estimates, sorted by portfolio size of CDFI, for real estate and business/consumer lending, undiversified 1% VaR, 2005



While total industry-level exposures have some relevance for policy decisions, the usual application of VaR is to inform risk management decisions at the institution level. The institution-level VaR estimates are not included in this report to protect the confidentiality of the firm-level information, but they are available to the CDFI Fund administrators. A template is shown below.

Template for VaR estimates of standalone risk							
Loan type	Loan amount (\$million)		Risk factor	Value at Risk (\$million)			
Home purchases		Х	0.12424				
Single family construction		Х	0.36843				
Multifamily construction		Х	0.16818				
Commercial real estate		Х	0.20321				
Other		Х	0.34324				

Portfolio balances categorized by type are entered in the second column. The loan amount is multiplied by the risk factor associated with that loan type. The right-most column is the estimate of undiversified VaR for each loan type, obtained by multiplying the loan amount times the risk factor. The Value at Risk column entries, by type of loan, are summed to obtain the total VaR from the lending activity during the month.

As an example of an institution-level calculation of VaR, consider the situation for a small CDFI with a diverse set of lending activities:

Example 1, VaR estimate of standalone risk						
Loan type	Loan amount (\$million)		Risk factor	Value at Risk (\$million)		
Home purchase	0.0409	Х	0.12424	0.0051		
Single-family construction	0.6280	Х	0.36843	0.2314		
Multifamily construction	0.0826	Х	0.16818	0.0139		
Commercial real estate	0.5492	Х	0.20321	0.0111		
Other	1.94e-08	Х	0.34324	6.66e-09		
Undiversified value at ris	0.2615					

Example 1 shows the undiversified VaR calculation for this small institution. The loan amounts for each type are multiplied by the corresponding risk factor and subsequently added to obtain the undiversified VaR of \$261,500. A larger CDFI in Example 2, an institution that is more focused on lending for home purchases and rehabilitation (no lending to commercial real estate projects or business), has a VaR that is estimated as follows:

Example 2, VaR estimate of standalone risk						
Loan type	Loan amount (\$million)		Risk factor	Value at Risk (\$million)		
Home purchase	1.2716	Х	0.12424	0.1580		
Single-family construction	0.3700	Х	0.36843	0.1363		
Multifamily construction	0.1872	Х	0.16818	0.0315		
Commercial real estate	0	Х	0.20321	0		
Other	0	Х	0.34324	0		
Undiversified value at ris	0.3258					

The CDFI shown in the next example has a larger portfolio, more than \$3 billion, which translates into a moderate VaR of \$397,000. All its loans are concentrated in lending for home purchases and rehabilitation. This CDFI is truly undiversified between loan types, and in this case the undiversified VaR estimate is the same as the diversified VaR estimate. Because homes are a relatively low-risk asset class, the risk is relatively low, even though the portfolio is not diversified.

Example 3, VaR estimate of standalone risk						
Loan type	Loan amount (\$million)		Risk factor	Value at Risk (\$million)		
Home purchase	3.1935	Х	0.12424	0.3967		
Single-family construction	0	Х	0.36843	0		
Multifamily construction	0	Х	0.16818	0		
Commercial real estate	0	Х	0.20321	0		
Other	0	Х	0.34324	0		
Undiversified value at ris	0.3967					

	Rural (n=104)		Not rural (n=62))
	1%	5%	1%	5%
Mean	4.44	4.13	3.60	3.33
Median	1.74	1.61	1.18	1.10
Min	0.0003	0.0003	0.003	0.003
Max	41.0	37.7	37.8	35.0
Std	7.05	6.55	7.22	6.67
Sum	461.7	429.8	223.1	206.4
Value at	Risk in real estate	(\$ million, undivers	ified)	
Mean	1.58	1.45	2.42	2.22
Median	0.21	0.19	0.34	0.32
Min	0	0	0	0
Max	39.61	36.43	29.77	27.52
Std	4.57	4.20	5.91	5.44
Sum	164.6	151.6	149.6	137.6

Table 11. Value at Risk (\$ million, undiversified), for CDFIs in rural vs. suburban and urban areas

Note: 14 CDFIs do not report rural or non-rural location and are not included in these estimates.

Extent of Risk Management through Diversification

Portfolio effects are considered to examine the possibility that CDFI managers are reducing risk through diversification among assets. Diversification benefits are observed for some institutions, but there is, overall, more evidence of increased risk due to portfolio effects (Table 12). The first columns in Table 12 are the summary statistics for the institutions, taking into account all interactions among assets, for the 180 CDFIs. On average, VaR is \$3.9 million (1 percent probability) (Table 12). The median 1 percent VaR is much lower, \$1.3 million, which is a characteristic of the many small CDFIs in the dataset. The large range between the highest and lowest VaRs is also a result of the sample including a few large institutions and many small ones. The largest VaR is \$36.6 million and accounts for 5 percent of the aggregate simulated portfolio VaR.

Overall and on average, the diversified portfolio VaR is larger than the estimate under the assumption that assets' returns are independent, which is shown in Table 13. This indicates increased risk due to the combination of assets in the portfolio. Portfolio VaR is \$18 million higher, industrywide (at the 1 percent level), due to dependence on loan types that are positively correlated. The industrywide finding is due to 62 individual CDFIs with estimated VaRs that have the unexpected portfolio effects.

While the number of institutions with higher risk estimates due to portfolio effects is just onethird of the total number of CDFIs, they are important institutions. VaR for the largest 20 institutions is depicted in Figure 9. The higher risk from portfolio composition can be seen by comparing the two bars. The solid bar is the diversified portfolio VaR; the striped bar is for the same large CDFI but is calculated assuming no asset interactions. The gaps between risk measures are in millions of dollars for the largest CDFIs, indicating the substantial contribution of large CDFIs to the unfavorable portfolio risk. Seven of the twenty largest CDFIs demonstrate risk reduction when diversification is taken into account (Figure 9). The size of the risk reduction for those seven large CDFIs is modest and overall does not outweigh the large risk increases that were found for the other large institutions. Among the 20 largest CDFIs, two have no difference in risk measures due to portfolio effects, because there is only one asset type. The average impact on risk due to lack of diversification benefits is \$122,551.

Among the smallest CDFIs, the total risk exposure is correspondingly small and any portfolio effects are also minimal. Little difference in risk exposure in dollar terms can be found when asset correlations are included in the VaR estimates (Figure 10). For six of the twenty smallest CDFIs, accounting for diversification leads to a smaller risk estimate than if asset interactions are ignored. This finding indicates that portfolio diversification by managers at the smaller institutions has contributed to risk management.

	Full portfolio effects (Diversified Portfolio VaR)		No interactio (Independen VaR)	on t Portfolio
	1%	5%	1%	5%
Across institutions:				
Mean	3.91	3.40	3.82	3.30
Median	1.31	1.51	1.28	1.12
Range	36.61	30.62	32.36	29.76
Standard deviation	6.67	5.73	6.39	5.44
Total 180 CDFIs	705	612	687	594
Value of higher risk due to diversification (mean)	0.123			
Total higher risk due to diversification for 180	22.06			
CDFIs				
Number of CDFIs with higher risk due to	62			
diversification				

Table 12. Summary statistics on Value at Risk in million dollars, estimated from portfolio simulation

Table 13. VaR industrywide (in million dollars, sum over 180 CDFIs), for 2005, three portfolio VaRs

	1% percentile	5% percentile
Diversified Portfolio VaR: Simulation with full variance-		
covariance matrix	705	612
Independent Portfolio VaR: Simulation with no covariance	687	594
Undiversified Portfolio VaR: Undiversified VaR with kernel tail	765	644



Figure 9. Effect of portfolio diversification on VaR estimates, 20 largest CDFIs, 2005





Relationship of VaR to CDFI Characteristics

CDFIs' unique circumstances contribute to the portfolio mix and potentially to their risk exposures. A quantitative analysis was developed to consider which observable characteristics of the CDFI relate to the institution's VaR. Portfolio size is closely linked with VaR, by definition, and we explored two alternative controls for size of the CDFI in analyzing the characteristics. Both approaches confirmed that VaR is positively related to size of the CDFI (Table 14). The portfolio size is linked with increasing VaR, but the marginal impact diminishes as the size goes up (column labeled Model 1, Table 14). This suggests some benefits in risk management are implemented as the portfolio is expanded.

An alternative framework to account for size of the CDFI is to categorize CDFIs by size groupings. Three groups were formed and binary variables were used to control for the size grouping. The results are reported in the column labeled Model 2 (Table 14). VaR is greatest for the largest institutions, not surprisingly, and the smallest group's VaR is less than the average VaR for the medium-sized CDFIs.

Having controlled for size of the CDFI, the other characteristics that were found to be related to riskiness of the institution are diversification, urban/suburban location, and minority-owned (Table 14). Diversification is associated with a reduction in risk, on average. We use the number of loan types in the portfolio of each institution as one indicator of diversification in this analysis. As the number of loan types increases, the associated average decline in the VaR is significant, estimated between \$160,000 and \$200,000.

The statistical finding of the benefits from diversification is explored further with variables that distinguish the impact of each type of loan. A high percentage of lending for home loans is strongly associated with lower risk. Likewise, a marginal increase of the portfolio's share in multifamily or retail real estate also contributes to a reduction in VaR. The mortgage lending category tends to be associated with higher risk exposure, but the impact is not significant, when contrasted with business lending.

Location in an urban or suburban area is associated with higher VaR, controlling for size, compared with the rural CDFIs. Ownership or control by minority group members is also associated with larger risk. The minority-owned CDFIs' VaR is on average \$347,000 greater than all other CDFIs in the sample. It should be noted that this estimate for minority-owned CDFIs was not robust across the methods to control for size. The type of financial institution and ownership by women or faith-based groups are not significantly related to the VaR.

Dependent variable: Value at Risk (\$1.000.000)	Model 1	Model 2
Total portfolio (\$1,000,000)	0.236***	
	(19.85)	
Square (total portfolio)	-0.0004***	
S June (com portions)	(-4.68)	
Size of portfolio (base = CDFIs with \$10-80 million	n portfolio)	
Institutions with \$80+ million portfolio		16.353***
-		(6.92)
Institutions with less than \$10 million portfolio		-3.993***
-		(-10.74)
Financial institution type (base = credit union)		
Bank or thrift-national	-0.277	1.394
	(-0.28)	(0.41)
Bank or thrift-state chartered	1.461	0.884
	(0.74)	(0.43)
Loan fund	-0.034	-1.550
	(-0.14)	(-2.46)**
Venture capital fund	-0.105	-1.849**
	(-0.32)	(-2.46)
Women-owned	-0.014	0.297
	(-0.07)	(0.70)
Minority-owned	0.347**	0.446
-	(2.09)	(1.24)
Faith-operated	-0.191	-0.160
-	(-0.53)	(-0.19)
Urban and suburban (base=rural)	0.333*	0.770***
	(1.97)	(2.79)

 Table 14. Results of regression analysis of VaR and CDFI characteristics,

 2005

Dependent variable: Value at Risk (\$1,000,000)	Model 1	Model 2
Number of loan types	-0.206*	0.161
	(-1.95)	(0.80)
	-	-
Share of type 1 (homes)	1.678***	1.530***
	(-6.42)	(-3.77)
Share of type 2 (mortgages)	1.083	-1.417
	(1.43)	(-0.76)
Share of type 2 (multifemily)	- 1 725***	-1.035*
share of type 5 (muthaniny)	(-5 59)	(-1.68)
	-	(1.00)
Share of type 4 (retail real estate)	2.012***	-2.317**
	(-3.15)	(-2.02)
Constant	0.635	5.616***
	(1.53)	(5.03)
Ν	161	161
R-square	0.963	0.864

Table 14—Continued. Results of regression analysis of VaR and CDFI characteristics

Figures in parentheses are t statistics of the estimated coefficients. Single, double, or triple asterisks (*, **, ***) represent significance at the 10%, 5%, and 1% levels, respectively.

Validation of the Findings

The validation steps include the following: (a) comparisons of VaR of CDFIs with their internal risk management indicators; and (b) comparisons between CDFIs' VaR and VaRs for other financial institutions.

Comparison of Value at Risk with Institutions' Internal Risk Management Indicators

For a first check on the reasonableness of the VaR estimates, data from the CDFI Fund institutions are used. Loan loss reserves (accrual and cash methods of accounting) are determined by CDFI management at many of the reporting institutions. For each institution,⁶ the reported reserves are compared with the estimates of VaR.

It stands to reason that CDFI managers are reserving adequately, to reflect average risk, but not reserving as to prepare for the worst-case scenario. As the worst-case risk indicator, we expect to find that reserves are below VaR, because reserves are set for ordinary risk management and not for a low probability-high loss event. The findings of the reserves analysis confirms these expectations.

There are two types of accounting procedures used to report loan reserves held by CDFIs, loan reserves by the cash method of accounting or by the accrual method of accounting. We select accrual method loan loss reserves as the internal risk management indicator to compare with VaRs for the following reasons: (a) 122 of the 180 CDFIs reported their loan loss reserves using the accrual method and 52 reported reserves using the cash method; and (b) CDFIs tend not to report loan reserves using the cash method. The four largest CDFIs, whose total portfolio exceeds \$80 million, reported only accrual method loan loss reserves.

Most of the institutions are holding reserves below the VaR estimated for the portfolio at 5 percent confidence. Only five CDFIs appear to be reserving in a highly conservative fashion, with reserves greater than the worst-case outcome predicted by the VaR. Fifty-six of the CDFIs did not report a loan loss reserve in 2005.

The implications of these comparisons between VaR and reserves highlight the trade-offs inherent in community development lending. A more conservative reserves policy to reach the level of VaR at 5 percent means less funding available for projects in the community. Increasing reserves to reach the VaR could reduce the CDFI's beneficial impact on the community.

For the few institutions that reserve in excess of VaR amounts, management may wish to consider whether relaxation of the reserve standards is feasible. Some managers' reserve decisions are linked to regulatory requirements that are not adjustable, and others may have reserved because of borrower-specific factors.

⁶ The CIIS data does not indicate the type of reserve measures used by all reporting CDFIs and there are no observations on reserves for several institutions. However, this does not necessarily mean that the 56 CDFIs that do not report loan loss reserves do not have such reserves.

The reserved amounts were analyzed for several classifications of CDFIs. The break-outs are for size and portfolio composition. Other classifications are meant to identify differences that may relate to special missions of the institution. Figure 11 contrasts loan reserves and VaR by different classifications for size and portfolio composition of the CDFI. The VaR estimates in the figures are the diversified portfolio VaR at the 5 percent level. The bars indicate the mass of observations, with dots indicating a few more dispersed observations. The detailed summary statistics that correspond with the figures are reported in Table 15.

Figure 11(a) is grouped by CDFI portfolio size: less than \$10 million (114 CDFIs), between \$10 million and \$80 million (56 CDFIs), and over \$80 million (10 CDFIs). The top bar (darker) for each group is the loan loss reserves and the lower bar (light) for the group is VaR. The lower bar for each group reinforces the scale dependence of VaR; larger institutions typically have more exposure when exposure is measured in dollars. On the top row of Figure 11(a), there are a number of small institutions, and VaRs are small. Reserves are roughly equivalent to VaR estimates. For the medium-size institutions, VaR is generally much greater than reserves. Larger institutions likewise typically have not reserved as much as the exposure measured with VaR.

Figure 11(b) groups CDFIs by the focus on real estate in the portfolio, compared with business/consumer lending. The intent of the groupings is to show the situation of specialized institutions that have a majority of portfolio value in real estate. There are 76 CDFIs with more than 50 percent of the portfolio in real estate lending. VaR is larger than the reserves in general, for both groups. The gap between reserves and VaR is roughly equal for both groups. This visually confirms the finding that real estate is not a uniquely large source of risk for CDFIs.

Figure 11(c) is composed of CDFIs that have at least 50 percent of the portfolio in real estate, and subsequently groups them by concentration across real estate loan types. The first group is heavily dependent on type 2 (mortgage lending), while the second group focuses on lending for home purchase or rehabilitation (type 1), multifamily housing (type 3), and retail development projects (type 4). The second group's real estate assets are less risky than the first group's. However, the small number of CDFIs in the risky real estate group is notable. Both groups have lower loan reserves than VaR, especially for group 1.

There are few significant differences in the total portfolios between rural vs. suburban and urban, woman-owned vs. others, minority-owned vs. others, and faith-based vs. others, as indicated in Figure 7 and in the regression analysis (earlier in the report). The same groups of CDFIs are examined to contrast the VaR exposure estimate with the institutions' loan loss reserves. CDFIs in the suburban and urban areas and women-owned CDFIs tend to have a much larger VaR than their internal loan reserves compared with their counterparts (see Figures 12(a) and 12(b)). VaR and loan reserves between minority-owned CDFIs and others, and between faith-based and others, do not show a significant difference. Even though bank or thrift-national CDFIs, bank or thrift-state chartered CDFIs, and credit union CDFIs have a much larger portfolio value (see Figure 7(e)), these CDFIs do not report any loan reserves (Figure 12(e)). The smaller CDFIs, namely loan fund and venture capital fund CDFIs, retain some loan loss reserves with amounts that are generally smaller than VaR.

We find that VaRs are reasonable representations of a conservative risk indicator, as judged by the fact that only five institutions reported provisions for loan loss in excess of the VaR exposures.

	No. of	Mean	Std.	Skew-	Kurtosis
	observa		Dev.	ness	
	-tions				
By portfolio size (\$1,000,0	(000				
< 10	79	0.24	0.30	2.71	11.98
< 80	39	0.85	0.74	1.37	4.20
80+	4	7.41	8.63	1.04	2.23
By real estate loan types					
Heavily on type 2	3	0.86	0.87	0.56	1.5
Heavily on types 1,3,4	59	0.85	2.69	6.54	47.00
By real estate or business					
Heavily on real estate	62	0.85	2.62	6.66	49.03
Heavily on business	60	0.48	0.66	2.48	8.72
Portfolio VAR (5%)					
By portfolio size (\$1,000,0	(000				
< 10	114	0.74	0.72	1.42	4.49
< 80	56	5.37	3.87	1.60	5.15
80+	10	22.67	5.27	0.18	1.52
By real estate loan types					
Heavily on type 2	3	7.53	9.64	0.67	1.50
Heavily on types 1,3,4	73	3.66	6.77	2.68	9.21
By real estate or business					
Heavily on real estate	76	3.81	6.86	2.54	8.46
Heavily on business	104	3.09	4.75	2.81	12.39

Table 15. Loan loss reserves of CDFIs, accrual method, by size of portfolio and by type of project (business or real estate)



(c) By real estate decomposition





(b) By the share of real estate portfolio



Figure 12. Loan loss reserves and VaR, by different types of institution

(e) By financial institution type

Value at Risk for CDFIs Compared with Other Financial Institutions

Another perspective on the VaRs estimated for CDFIs is gained by comparison with the related risk measures reported by some of the largest banks in North America. This comparison confirms the validity and practicality of using VaR as well as its method of implementation in

large firms. We expected, and subsequently found, very few similarities in the dollar values for VaR for large banks and CDFIs due to the relatively small size of CDFIs relative to global banks.

The eight largest money center banks in North America report a VaR measurement in their 2007 annual 10-K reports (Securities and Exchange Commission). These large banks use VaR to monitor their exposure to trading losses in various sectors of the capital markets. The portfolios of money center banks consist of highly volatile assets traded daily, in markets in which many small lenders are not involved. They calculate their VaRs based upon the 1 percent probability of possible losses on a *daily* basis.⁷ Some of the most common risks are foreign exchange, interest rate, credit derivatives, real estate, equities, and commodities. Figure 13 illustrates each bank's exposure in the different areas.

The undiversified VaRs, when converted to a monthly exposure estimate to be comparable to the procedures used for CDFIs, are substantially larger than those for the CDFIs. The VaR estimates are sensitive to scale of the portfolio, and therefore it is to be expected that the dollar levels will be substantially larger. The smallest VaR among the large banks is \$33 million (monthly basis), quite close to the largest CDFI's VaR.

When diversification is considered, market-based trading portfolio VaRs are much lower than if the VaR calculation ignores interaction among asset returns (Figure 14). For CDFIs, a similar result on the direction of benefits from the use of portfolio diversification to manage risk was found for many, but not all, of the institutions. Sixty-two CDFIs have VaR estimates that indicate no risk reduction from the portfolio.

The range of the benefits of diversification is from 27 percent to 52 percent of the undiversified VaR for the leading publicly traded financial institutions. The benefits of diversification found from the large banks' VaRs are likely due to the particular asset mix in these portfolios. In addition, the method by which diversification benefits are calculated can lead to different estimates. For example, in calculations that assume returns are normally distributed, diversification benefits may appear much larger than those from a historical simulation.

⁷ Daily x $\sqrt{30}$ = monthly.

Figure 13. Total market-based trading portfolio value at risk, excluding diversification effects, for leading publicly traded financial institutions, based on 1% daily exposure



*Wells Fargo does not separate VaR by asset type or without diversification. Source: Bank of America (2006); J.P. Morgan (2006); Citigroup (2006); Wells Fargo (2006); Wachovia (2006); Royal Bank of Canada (2006); Bank of New York (2006); Toronto Dominion (2006).

Figure 14. Total market-based trading portfolio Value at Risk, for leading publicly traded financial institutions, accounting for diversification effects



Source: Bank of America (2006); J.P. Morgan (2006); Citigroup (2006); Wells Fargo (2006); Wachovia (2006); Royal Bank of Canada (2006); Bank of New York (2006); Toronto Dominion (2006).

Relevance of Value at Risk to Managers

Stakeholders need to understand the extent to which the CDFIs are exposed to loss, which is difficult when those assets are not traded during a particular period of time. In this study, Value at Risk is shown to be an option for measuring and managing exposures associated with particular types of loans. In this section, we describe the practical applications of Value at Risk as a complement to existing risk management activities.

Lenders' risk management activities vary across institutions, depending on regulatory requirements, institutional missions, and individual managerial choices. Three essential activities form the basis of most risk management programs:

- Assessing risk levels, in terms of the probability of default.
- Monitoring loan exposure, that is, loss in the event of default.
- Communicating risk exposure within institutions and to other parties.

CDFI managers evaluate each loan for financial soundness individually, both upon execution of the loan and periodically throughout the loan term. A case-by-case evaluation of project-specific risk allows the management to consider a variety of factors that describe expected risk and reward from that loan. On the risk side, the borrower's expected performance is often assessed with indicators specific to the borrower, such as cash flow, collateral, and compliance with loan covenants. Investor requirements may lead a CDFI to include other indicators as well. Each loan is then assigned a risk rating. Risk ratings are weighed against the contributions of the project to the institutions' goals for financial sustainability, and/or the expected impact on community economic development.

While case-by-case risk ratings are the foundation of portfolio risk management, there is an important role for indicators that can communicate an overall picture of the institution's portfolio risk. One summary indicator that is currently in use at CDFIs is the loan portfolio quality indicator called Portfolio at Risk. It is measured as the percentage of the portfolio value that is delinquent, as determined by days past due.

The VaR measures presented in this study are distinct from the Portfolio at Risk delinquency indicator, in that VaR is an exposure of the entire portfolio based on historical returns of an asset class and is not linked to any particular borrower. Because VaR does not require management to wait for problems in the borrowers' performance to be detected, as delinquency measures do, VaR is a more proactive risk indicator in some respects. But because all loans in the portfolio category are assigned equal riskiness to the proxy variable for an asset class, VaR does not incorporate all the precise case-specific circumstances of the credit. Therefore, VaR is suited for broader oversight, regular updates, and for certain portfolio balancing questions and cannot substitute for lenders' actions to monitor or assist borrowers. Suggested uses for VaR among CDFI managers include:

• risk communication,

- loan loss reserve policy,
- credit concentration limits, and
- collateral valuation.

Risk Communication

The terms in which VaR is reported, as a dollar value, are more communicative of risk to most stakeholders than are other indicators reported by financial managers. As such, VaR will be particularly useful for managers to communicate with boards or community agencies that include volunteers having a variety of expertise in community development. For example, CDFIs work closely with conventional financial institutions, by means of formal or informal systems of client referral and partnership lending. In this context, VaR could be a way for CDFIs to compare and communicate with their counterparts in more-conventional financial institutions.

The perspective on the worst-case scenario is useful when economic conditions are strong, in that risks are framed in a context that is most conservative. A conservative outlook is useful during times when risks are most likely to be overlooked. When portfolio managers look at the detailed conditions of a proposed project in a favorable market, there is the natural potential for optimism. Particularly when qualities of the borrower and other important intangible factors are considered, it is possible for a portfolio manager to under-reserve. By definition of Value at Risk, the exposure estimate is grounded in the historical record for the greatest loss, and will be a contrasting perspective to the detailed loan rating.

When economic conditions are more difficult, as in recent months, the worst-case exposure estimate may already be at the forefront of the portfolio managers' minds. In these times, VaR may be used directly in decisions about loan loss reserves.

Role of VaR in Loan Loss Reserve Analysis

Managers can use the parameters from the underlying risk factors estimated in this study to update the CDFI's VaR on a regular basis. This step is recommended as part of the monthly portfolio review, or more frequently. A template and an example are shown below.

New loans closed, categorized by type, are entered in the second column (shaded). The loan amount is multiplied by the risk factor associated with that loan type. The right-most column is the estimate of undiversified VaR for each loan type, obtained by multiplying the loan amount times the risk factor. The Value at Risk column entries, by type of loan, are summed to obtain the total VaR from the lending activity during the month.

Template for updating VaR estimate of standalone risk						
Loan type	Loan amount (\$million)		Risk factor	Value at Risk (\$million)		
Home purchases		Х	0.12424			
Single-family construction		Х	0.36843			

Multifamily construction	Х	0.16818	
Commercial real estate	Х	0.20321	
Other	Х	0.34324	
Undiversified value at risk :			

For example, a CDFI's VaR is estimated at \$6.33 million, at the .01 percentile. Suppose this CDFI issues new loans in a particular month in four of the five loan types. How could its exposure estimate be updated? In a given month, suppose that new loans are being considered in roughly the same asset classes as its current portfolio. The template is filled in as follows, to estimate VaR associated with the lending activity during the month. This approximation does not take into account the interaction of the portfolio risk factors; therefore it is an estimate of standalone or undiversified VaR.

Example of updating VaR estimate of standalone risk						
Loan type	Loan amount (\$million)		Risk factor	Value at Risk (\$million)		
Home purchases	0	Х	0.12424	0		
Single-family construction	0.0594	Х	0.36843	0.022		
Multifamily construction	0.55	Х	0.16818	0.093		
Commercial real estate	2.33	Х	0.20321	0.467		
Other	0.44	Х	0.34324	0.151		
Undiversified value at risk:	0.734					

A worst-case estimate of the incremental exposure from the loans issued during the month is \$734,000. The estimate is fairly easy to develop and can be calculated in advance of completing the loans, or as part of monthly monitoring of a complex loan portfolio. Managers may choose to add this dollar value of exposure to reports to stakeholders, or for internal decision-making purposes, along with the other risk indicators that are routinely provided. A manager of a large institution will benefit from having this snapshot view of risk exposure with VaR on a frequent basis. Revising the estimates of aggregate VaR for the portfolio is simpler than a complete review of each loan in a large institution. If there are several loan officers or many loans considered, as typically required in the larger institutions, the VaR estimate is a useful supplement to case-by-case risk ratings. Alternative estimates of standalone VaR are easily obtained with this template, as long as the risk factors remain acceptable proxies.

We recommend that the CDFI boards receive a VaR estimate each month as a means to re-assess reserve levels. We do not propose that VaR estimates replace existing loan reserve decision-making processes. However, to the extent that a CDFI's reserves on the books exceed the total VaR, then management should consider whether the reserved amounts are too conservative. Because VaR is the worst-case estimate of exposure, in most instances, management would not need to reserve against that very special bad case.

Institutions that do not currently report reserves can use VaR estimates as a starting point in developing a more formal loan loss reserves program. The template shown above, when used with the total portfolio levels, will provide an upper bound on the benchmark for reserves.

Concentration of Credit

Portfolio managers often aim to limit concentration in any one industry, to assist in reaching the goal of diversification. Value at Risk estimates can assist portfolio managers along these lines, by providing additional information about riskiness of a particular sector. We propose that VaR estimates be used to supplement the concentration limits. Specifically, concentration in terms of total credit can be supplemented with concentration in terms of VaR levels. Consider an example:

A CDFI has \$26.8 million in loans for home purchase and rehabilitation. The other major category of lending is business/consumer loans (\$33.5 million). Concentration in real estate, in percentage terms, is high (44 percent). Compare this with the concentration of VaR. Using the simplest calculation of VaR, the undiversified VaR or the standalone risk from the loans is:

Example of VaR used to assess concentration of credit						
Loan type	Loan amount (\$million)	nount Risk ion) factor		Value at Risk (\$million)		
Home purchases	26.8	Х	0.12424	3.3		
Single-family construction	0	Х	0.36843	0		
Multifamily construction	0	Х	0.16818	0		
Commercial real estate	0	Х	0.20321	0		
Other	33.5	Х	0.34324	11.5		
Total	60.3			14.8		

As a result of the differences in risk profiles of the asset classes, managers can determine that the concentration in real estate is not as risky, as measured with VaR, as it would be according to the original concentration ratio. Loss exposure concentrated in real estate is only 22 percent, rather than the 44 percent concentration of total credit. In this example, VaR can be used as support for a decision to waive a credit concentration limit.

Collateral

Managers routinely weigh the risk rating of a project against the collateral that is pledged to secure repayment of the loan. To the extent that variability in the real estate markets undermines the value of collateral, then the series analyzed in this study are relevant to the risk-rating process undertaken by CDFI management. The market analysis of real estate asset classes provided by this study is relevant for understanding the collateral position of CDFIs, and therefore is an input into the risk-rating process for new credits. While risk profiles for asset classes within real estate can be differentiated with the price indexes used in this study, the indicators are based on national-level statistics and are not tailored to the specific local market conditions of each CDFI.

Limitations and Future Research

Any forward-looking risk indicator that is based on historical data is built with several limiting assumptions. Because the methods use a flexible probabilistic approach to describe the risk profile, the results presented in this study do not depend on a particular mathematical form, and are a reliable description of what has actually happened over a long period. Nevertheless, when extrapolating past risk to a risk management program, there is a possibility that markets will diverge dramatically from history and the actual realized loss may be different from the exposure estimate.

Notwithstanding the accuracy of the risk measures, it is important that the findings from the study not overstate the risk side of the risk–reward balancing that portfolio managers perform. Some risky ventures are undertaken because, on average, one can obtain higher rewards by bearing risk. The Value at Risk estimates shown here do not take into account the returns earned by the CDFI in association with its willingness to supply credit to borrowers in the community. Those returns may be tangible income for the institution as well as community-wide benefits in the form of employment or capacity building among the CDFI clients.

The risk exposures presented in this report have not been compared with the unique programs that CDFIs offer to assist borrowers in avoiding default, such as homebuyer education services. These activities are integral parts of some CDFIs' asset-building strategies and may result in risk reduction.

The risk exposure of CDFIs is affected by other factors that this study did not consider. Our focus has been on measurement of the common shocks in the market, using readily accessible information that reflects national aggregates. Local conditions and trends may well have been averaged out in the aggregation to the national level. To the extent that CDFIs are locally focused, the VaR estimates may understate risk because the proxy is based on a geographically dispersed indicator. Although Furlong and Krainer (2007) found only mixed evidence that average community bank performance is closely tied to the regional economy, further study of this issue specifically focused on CDFIs may be warranted. An option to study this issue further would include the following steps:

- Examine transaction-level data to identify the extent to which the CDFI concentrates the portfolio in a geographic area;
- and use regional indicators of business performance or real estate sector returns.

While more research will add to the ability of risk managers to refine their VaR estimates, key conclusions can be drawn from the detailed risk profiles for real estate and other lending estimated in this study. First, the exposure of the CDFIs to risks related to real estate in 2005 was substantial, \$350 million for 180 reporting CDFIs. Real estate did not dominate portfolio risk in the aggregate, yet some of the largest CDFIs have a high concentration of portfolio risk in real estate. The special circumstances of those CDFIs that have dependence on the real estate sector may require adjustments in risk management or planning for special assistance during the downturn in the housing market. Second, the management and policy recommendation that follows from this research is to report risk indicators for each CDFI using Value at Risk, allowing stakeholders to remain more informed about risk exposure.

Glossary of Terms

Assets – Something valuable that an entity owns, benefits from, or has use of, in generating income

Balance sheet – Condensed statement that shows the financial position of an entity on a specified date

Black Monday – Monday, October 19, 1987, when the Dow Jones Industrial Average fell by 508 points

Capital – Measure of the accumulated financial strength of an individual, firm, or nation, created by sacrificing present consumption in favor of investment to generate future returns above investment costs

Correlation – Degree and type of relationship between any two or more quantities (variables) in which they vary together over a period

Deflator – Rate of inflation used as a divisor to convert (deflate) retail prices or wages from nominal amounts to real (adjusted for inflation) amounts

Equity – Ownership interest or claim of a holder of common stock and some types of preferred stock of a firm

Mean – The average of a series of numbers

Nominal – Data series as quoted in the current values, without an adjustment for the effects of inflation

Parametric – A variable entering into the mathematical form of any distribution such that the possible values of the variable correspond to different distributions

Percentile – The p^{th} percentile (for p between 0 and 100) is the observation such that approximately p percent of the observations are less than this observation and 100 - p observations are greater than this observation

Probability density function – A function of a continuous random variable whose integral over an interval gives the probability that its value will fall within the interval

Quantile – A quantile is the observation such that a fraction of the observations are less than this observation (e.g., the 100 quantiles are called percentiles, see the definition of percentile)

Rate of return – The ratio of money gained or lost on an investment relative to the amount of money invested

Real Estate Investment Trust (REIT) – A security that sells like a stock on the major exchanges and invests in real estate directly, either through properties or mortgages **Skewness** – Asymmetry in a probability distribution

Small-cap – U.S. stock with market value below \$500 million

Standard deviation – Measure of the unpredictability of a random variable, expressed as the average deviation of a set of data from its arithmetic mean and computed as the positive square root of the variance

Standard normal distribution – A distribution with a mean of zero and a variance of one

Appendix A: Results When Returns Assumed Normally Distributed

Using the current portfolio value v^0 as the initial investment, the random rate of return *R* defines the future value for the risky asset, *V*. At time 1, the portfolio value is:

(A-1) $V^1 = v^0 (1+R).$

R is a random variable having expected value μ and volatility σ . VaR measures focus on a tail loss probability of *V*. The probability associated with the loss outcome (*c*) will be selected at 0.05 or 5 percent worst outcome, so that our result will describe the worst-case scenario in which 95 percent of the possible outcomes are better than the tail loss.

The tail loss outcome for *R* is defined as R^* and is simply derived from the lower tail of the empirical distribution of asset returns. Using equation (A-1), the predicted portfolio value upon realization of the worst case, V^* , is calculated. The VaR is presented as the difference between V^* and the expected value of V^I (relative VaR).

For assets having a distribution that fits well with the standard normal distribution, statistical tables are used to obtain VaR using the following formula for the cumulative standard normal:

$$1-c=\int_{-\infty}^{-\alpha}\Phi(\varepsilon)d\varepsilon\,,$$

where c is the selected quantile for lower tail loss. We use c = 0.05 and C = 0.01 as comparisons.

While it is common to assume normality when calculating portfolio VaR, it is important to understand the consequences of making such a simplifying assumption. The summary statistics and histogram indicate that by making the assumption of normality, one must ignore the skewness of the data. Further considerations that focus on the tails of the distributions are shown in Figure A-1, the quorm plots that plot the quantiles of a random variable against the quantiles of the normal distribution. The solid red line is the 45-degree line. In principle, this random variable follows a normal distribution if all the observations are on the 45-degree line. In general, the greater spread of the extreme quantiles for the data is indicative of a fat tail distribution.

Arguably, the rate of return for the new-home sales median price (type 1) fits the normal distribution better than other proxies, and the rate of return for the mortgage REIT basket (type 2) is likely to be non-normal. Formally, we conduct various tests for normality and present test results in Table A-1. The null hypothesis that the rate of return follows a normal distribution was rejected for types 2, 4, and 5 at the 1 percent significance level. The null was not rejected for types 1 and 3. The results suggest that the rate of return for type 1 or type 3 may follow a normal distribution, but the distribution of the rate of return for other types is not normal.

Two alternative statistical procedures are conducted to derive the tail loss of the rates of return for five loan types: one that assumes that rates of return on assets follow a normal distribution, and another that is nonparametric. Under the assumption that asset returns are normally distributed, the estimated tail losses at a 1 percent confidence level for the real estate assets range from 9 percent to 20 percent. Percentage loss estimates are correspondingly lower when the 5 percent confidence level is selected.



Figure A-1. Plots to compare the distribution of the real estate and business assets with the normal distribution.

While the assumption that returns follow a normal distribution is convenient, the possibility exists that "...a model based on a normal distribution would underestimate the proportion of outliers and hence the true value at risk" (Jorion 2001, 221). The true distributions of returns on assets are often observed to have fat tails compared with the smoothly declining tail area under a normal distribution. For every asset type, the normal assumption results in a lower estimate of tail loss percentage. The loss estimates under the empirical distribution range from 12 percent to 36 percent, representing a worst-case outcome for a month. The gap between the two procedures is particularly noticeable on asset type 5, business and consumer loans. Because many of the CDFIs are involved in type 5 projects, the final value-weighted estimate of VaR is substantially higher under the empirical procedure than when a normal density is assumed (Table A-2 and A-3).

Loan type	Skewness-and	Shapiro-Wilk test	Shapiro-Francia	Jarque-Bera test
	kurtosis test		Test	
Type 1	0.02	0.65	0.35	0.07
	(0.99)	(0.26)	(0.36)	(0.96)
Type 2	17.71	4.21	4.11	50.82
	(0.00)	(0.00)	(0.00)	(0.00)
Type 3	2.93	1.07	1.13	2.54
	(0.23)	(0.15)	(0.13)	(0.28)
Type 4	9.82	2.37	2.58	19.60
	(0.01)	(0.01)	(0.00)	(0.00)
Type 5	52.34	5.46	5.12	263.75
	(0.00)	(0.00)	(0.00)	(0.00)

Table A-1. Normality test results of the rate of returns for five loan types

Note: Figures in parentheses are the corresponding *p*-value, and figures above parentheses are test statistics for each test. The references for these four different normality tests are given below: Skewness-and kurtosis test: D'agostino et al. (1990);

Shapiro-Wilk test: Shapiro and Wilk (1965);

Shapiro-Francia test: Shapiro and Francia (1972); and

Jarque-Bera test: Jarque and Bera (1980).

		Average across institutions				
	Sum (180 CDFIs)	Mean	Std dev	Min	Max	
1% portfolio VaR	295.63	1.64	3.00	0.00	17.11	
5% portfolio VaR	209.03	1.16	2.12	0.00	12.10	
1% portfolio VaR, 0 covariance	269.42	1.50	2.63	0.00	16.49	
5% portfolio VaR, 0 covariance	190.49	1.06	1.86	0.00	11.66	
1% undiversified VaR	347.67	1.93	3.57	0.00	22.46	
5% undiversified VaR	245.82	1.37	2.53	0.00	15.88	

Table A-2. Value at Risk estimates, in million dollars, for CDFIs assuming normal distribution of asset returns

Table A-3. Value at Risk, undiversified, kernel density estimation compared with normal distribution (in million dollars)

	Sum (180 CDFIs)	Mean	Std dev	Min	Max
Normal 1%	347.67	1.93	3.57	0.00	22.46
Normal 5%	245.82	1.37	2.53	0.00	15.88
Empirical 1%	765	3.85	6.60	0.00	35.73
Empirical 5%	644.17	3.58	6.12	0.00	32.88

Appendix B: Kernel Density Estimation

Kernel density estimation is an empirical method for estimating the probability density function of a random variable. Given a data sample of a population, kernel density estimation extrapolates the data to the entire population.

Let $K(\cdot)$ be a smooth probability density function, called the kernel function. Suppose we have a random sample of N observations of the rates of return r_t , r_1, r_2, \dots, r_T . Using the kernel method, the probability density of r_t at a given point x, denoted by f(x), is estimated using the equation:

$$\hat{f}(x) = \frac{1}{N} \sum_{i=1}^{N} K \left(\frac{x - r_i}{h} \right),$$

where h > 0 is the bandwidth (Hardie 1990). There are various kernel functions, including the uniform, triangle, Epanechnikov, Quartic, Triweight, Cosinus, and Gaussian kernel functions. Silverman (1986) suggests that the choice of a kernel function has minimal effect on the density estimates. In this project we choose $K(\cdot)$ to be the density function of the standard normal random variable:

$$K(\mu) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\mu^2\right)$$

It is well known that the bandwidth h is critical in estimating the probability density function. Small values of h lead to very spiky estimates (not much smoothing), while larger values of h lead to oversmoothing. A common method for determining the optimal bandwidth is to use the bandwidth that minimizes the asymptotic mean integrated square error (Sheather and Jones 1991). Silverman (1986) also suggests that the optimal size of h can be cross-validated.

A histogram can be thought of as a collection of samples from a kernel density estimate for which the kernel is a uniform box with the width of the histogram bin. Intuitively, histograms group observations into bins. The histogram density estimator can be asymptotically consistent but less smooth. The kernel density estimator can be thought to have small "bumps" at each observation that are determined by the kernel function. The kernel density estimator consists of a "sum of bumps" and is smoother than a frequency histogram.

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